MODULE *Maths*1

EXTENDS *Integers*

$p(x) \triangleq x^2 - 3 * x + 2$

THEOREM $A \triangleq$ ASSUME NEW $x \in Int$
PROVE $x = 1 \Rightarrow p(x) = 0$
PROOF BY DEF p

THEOREM $B \triangleq \neg(\forall\, x \in Int : p(x) = 0 \Rightarrow x = 1)$
PROOF BY $p(2) = 0$ DEF p

THEOREM $C \triangleq \neg(\forall\, x \in Int : p(x) = 0 \equiv x = 1)$
PROOF BY B

THEOREM $D \triangleq \forall\, x \in Int : p(x) = 0 \equiv (x = 1 \vee x = 2)$
$\langle 1 \rangle$ TAKE $x \in Int$
$\langle 1 \rangle 1.\ p(2) = 0$ BY DEF p
$\langle 1 \rangle 2.\ x \neq 1 \wedge x \neq 2 \Rightarrow p(x) \neq 0$ BY DEF p
$\langle 1 \rangle 3.$ QED BY $\langle 1 \rangle 1$, $\langle 1 \rangle 2$, A

THEOREM $E \triangleq \neg(\forall\, x \in Int : p(x) = 0 \equiv (x = 1 \vee x = 2 \vee x = 3))$
PROOF BY D

THEOREM $F \triangleq \neg(\forall\, x \in \{1, 2, 3\} : p(x) = 0)$
PROOF BY DEF p